

The role of thermal properties in periodic time-varying phenomena

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Abstract

The role played by physical parameters governing the transport of heat in periodical time-varying phenomena within solids is discussed. Starting with a brief look at the conduction heat transport mechanism, the equations governing heat conduction under static, stationary and non-stationary conditions, and the physical parameters involved, are described. Special emphasis is given to the phenomenon of diffusion of heat in the presence of periodical heat sources and the related (still unknown for the majority of people) thermal effusivity concept.

1. Introduction

The conduction of heat in solids is a well-known phenomenon, whose mathematical description dates from about hundred years ago, when Fourier [1] stated his famous law. He also showed that expanding temperature distributions as a series of waves could solve heat conduction problems, anticipating the concept of the thermal wave. These have become of great interest in the explanation of the photoacoustic (PA) effect and other photothermal (PT) phenomena, on which several measurement techniques are based [2]. In PT techniques, the periodic absorption of energy without re-emission losses leads to sample heating, which at the same time induces changes in temperature-dependent parameters of the sample itself and/or of the surrounding medium, whose detection is the basis of different experimental methods. Because in these methods the temperature changes follow the periodical modulation of the excitation light beam, they are often interpreted as thermal waves and many authors have adopted the principles of wave physics to interpret data from PA and PT experiments. However, the role played in these phenomena by the thermal parameters governing heat transport is often not well known or is misinterpreted. Therefore, it is the purpose of this work to discuss some

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aspects of thermal transport in the case of periodic heat sources emphasizing physical aspects related to the physical parameters involved, in particular, the thermal effusivity.

On the other hand, as thermal properties are key parameters governing the behaviour of many processes in nature, it is of great importance to deal with this theme with college or university level science and engineering students, as well as develop experiments for their measurement and illustrate their physical significance in different phenomena. From the several methods that should be used for this purpose, the PT techniques offer some advantages that make them suitable for student laboratories, such as the relatively simple and inexpensive (as possible) equipment required and understandable physical–mathematical formalism for a given educational level. In this paper, we will also describe some experiments that can be useful for introducing students to the basic ideas and principles of these methods and of the physics behind periodical, time-varying heat transfer phenomena.

This paper is organized as follows. In the next section, a brief look at the stationary conduction heat transfer mechanism will be given, with emphasis on the concept of thermal conductivity. In section 2, the case of conduction of heat under non-stationary circumstances and the physical significance of the thermal diffusivity and specific heat will be described. In the particular case of periodical heat sources, thermal effusivity will be defined and its physical significance exemplified. In section 4, some historical remarks and the mathematical basis of thermal wave physics as well as some experiments demonstrating the validity of the wave description of the conduction of heat in the presence of periodical heat sources will be described. The conclusions will be outlined in section 5.

2. Conduction heat transport

It is well known that any temperature difference within a physical system causes a transfer of heat from the region of higher temperature to that of lower. This transport process takes place until the temperature of the system becomes uniform throughout. Thus, the quantity of heat, H , transferred per unit time, t , should be some function, Φ , of the temperatures, T_1 and T_2 , of both the regions involved, i.e.,

$$\frac{\partial H}{\partial t} = \Phi(T_1, T_2). \quad (1)$$

It is denoted the rate of heat flux (units of W) and its form depends on the nature of the transport mechanism, which can be radiation, convection or conduction (or a coupling of them) [3]. Radiation is the continuous energy interchange between separated bodies by means of electromagnetic waves while convection takes place by means of macroscopic fluid motion. It can be caused by an external source (forced convection) or by temperature-dependent density variations in the fluid (free or natural convection) [4]. The conduction of heat is a process where the thermal energy, in the case of solids, is mainly transported by phonons, the quanta of lattice vibrations, and/or free electrons. In the case of fluids (i.e. liquids and gases) this process occurs through the movement of atoms and molecules in a more complicated form due to the presence of other hydrodynamic effects, such as the heat convection described above. In this paper, we will focus our attention on heat transfer by conduction.

2.1. Fourier's law and thermal conductivity

Thermal conduction can be understood as a microscopic down-temperature diffusion process of heat within solids and stagnant fluids. The local heat flow rate in some direction, r , of a homogeneous material is governed by Fourier's law:

$$\Phi_{\text{cond}} = -kA\nabla T. \quad (2)$$

The thermal conductivity, k ($\text{W cm}^{-1} \text{K}^{-1}$), is expressed as the quantity of heat transmitted per unit time, t , per unit area, A , and per unit temperature gradient $\nabla T = \partial T / \partial r$. The negative sign indicates that heat flow will take place in the opposite direction of the temperature gradient.

Thermal conductivity is a measure of the ability of a material to conduct heat. It characterizes stationary (not static) heat transfer problems. Although it depends on temperature as well as pressure and for solids it is a highly structure-sensitive parameter, over moderate ranges of temperature and pressure, k can often be considered as constant. For one-dimensional steady-state conduction in extended samples of homogeneous and isotropic material, Fourier's law can be integrated in each direction to its potential form. In rectangular coordinates it reads

$$\Phi_{\text{cond}} = -kA \frac{T_2 - T_1}{x_2 - x_1}. \quad (3)$$

Here k is called the mean thermal conductivity. T_1 and T_2 represent two planar isotherms at positions x_1 and x_2 , respectively. Due to its analogy with electrical conduction, equation (3) is often defined as Ohm's law of thermal conduction [3].

3. Non-steady heat conduction

3.1. Thermal diffusivity and specific heat capacity

When a material is subjected to non-steady heating or cooling, its inner temperature profile is given in terms of time and spatial position. The resulting conduction can be analysed by combining Fourier's law with a heat flow balance (energy conservation law) [5]. Assuming constant thermal conductivity this leads to a parabolic heat equation, often called Fourier's second law:

$$\nabla^2 T - \frac{1}{\alpha} \frac{\partial T(r, t)}{\partial t} = -\frac{Q}{k}, \quad (4)$$

where ∇^2 is the Laplacian operator. For one-dimensional heat flow, this equation becomes

$$\frac{\partial^2 T(x, t)}{\partial x^2} - \frac{1}{\alpha} \frac{\partial T(x, t)}{\partial t} = -\frac{Q}{k}. \quad (5)$$

Here Q ($\text{J cm}^3 \text{s}^{-1}$) denotes possible heat losses ($Q < 0$) or generation ($Q > 0$) per unit volume per unit time. The coefficient α ($\text{cm}^2 \text{s}^{-1}$) represents the thermal diffusivity, the rate at which a temperature variation propagates through the material. In the next section we will refer in more detail to the significance of this parameter, which can be defined as

$$\alpha = k/C, \quad (6)$$

where C ($\text{J cm}^{-3} \text{K}^{-1}$) is the specific heat capacity, or heat capacity per unit volume of the material, defined as the product of density, ρ (g cm^{-3}), and specific heat, c ($\text{J g}^{-1} \text{K}^{-1}$):

$$C = \rho c. \quad (7)$$

Specific heat is the amount of heat that is required to raise the temperature of a unit mass of a substance by one degree, characterizing static problems in heat transfer.

Thermal diffusivity can be considered, therefore, as the ratio of the heat conducted through the material to the heat stored per unit volume.

Usually one refers in the above equations to the specific heat at constant pressure because the difference between that value and those at constant pressure (due to the expansion energy) is small, at least for solids [6].

If we look at the tabulated values for ρ and c given in the literature we notice that large differences exist between different materials. The cause is, essentially, that the specific heat is

linked to the internal energy (such as rotational and vibrational energy) of a molecule. More correctly, as mentioned above, the specific heat is the change in the internal energy per unit of temperature change. The more degrees of freedom the molecule has, the more energy it can store. Now suppose we have a material with a high density, which means that the molecules are very close to each other. Certainly this reduces the ability to vibrate. In other words, the material has a low specific heat. The result is that the product $C = \rho c$ falls within a relatively narrow margin [3].

It is worth noting again that the specific heat, characterizing (as explained above) the material's ability to store heat, describes static problems, where temperature is independent of time and position, whereas the thermal conductivity is the parameter required when leading with steady problems, i.e. when the temperature does not vary over time, as described by Fourier's law (equation (2)) of heat conduction. Thermal diffusivity, on the other hand, describes non-stationary phenomena, where temperature depends on both time and position as given by equation (4).

3.2. Periodic heat sources: thermal effusivity

Another important (although a rather unknown quantity in physics) thermal parameter is the so-called thermal effusivity² ($\text{W s}^{1/2} \text{cm}^{-2} \text{K}^{-1}$), named also 'thermal admittance' and 'contact coefficient' by some authors [7]. It is defined as

$$\varepsilon = \sqrt{k\rho c} = k/\sqrt{\alpha} = \rho c\sqrt{\alpha}. \quad (8)$$

It is a relevant thermophysical parameter for surface heating or cooling processes [2], as well as in the recent developed field of thermal wave interference physics [8–12], where it determines the value of the reflection and transmission coefficients for thermal waves propagating at the interface between two media (it is a measure of the thermal mismatch between them) as will be described later. Part of the objective of this work will be therefore the discussion about its meaning and its role in transient and time-varying heating phenomena.

3.2.1. Transient phenomena: 'filling' temperature. Our scientific understanding of heat was developed during the last two hundred years, but problems like the following are still with us. Sensory experience is the way in which we interact and measure the world around us, and we (teachers) face several problems when the conclusions we draw from this experience conflict with accepted interpretations and theories. One often-asked question is about the preferred explanation for a situation such as the following: *if students were presented with objects made of metal and wood, and they felt the objects with the palms of their hands, then they would think that the wooden object feels hottest*. Many people suggest that the described effect is due to the fact that *metal conducts energy away from ones hand more rapidly than wood* [13]. This assertion leads to the mistaken notion that the relevant thermophysical parameter for the phenomenon described is the better known thermal conductivity instead of the thermal effusivity.

To demonstrate that thermal effusivity is the relevant parameter when the contact of human skin with a material at different temperatures is studied, we can start the analysis considering a semi-infinite homogeneous medium experiencing (uniformly, i.e. in such a way that the one-dimensional approach used in what follows is valid) a sudden temperature change at the surface from T_0 to T_1 , e.g. heating a material (a thermal reservoir) initially at temperature T_0 by (perfect thermal) contact with human body skin at a higher temperature T_1 . For the calculation

² Many authors use the symbol $(k\rho c)^{1/2}$ instead of ε .

of the temperature evolution in the material medium, one has to solve the homogeneous heat diffusion equation

$$\frac{\partial T(x, t)}{\partial t} = \alpha \frac{\partial^2 T(x, t)}{\partial x^2}, \quad (9)$$

with the boundary conditions

$$T(x = 0, t \geq 0) = T_1; \quad T(x > 0, t = 0) = T_0. \quad (10)$$

These equations imply that the body skin temperature T_1 remains constant in the course of time. At the material/human skin interface a heat flow will start. The solution of equation (9) with these conditions leads to [7, 14]

$$T(x, t) = T_1 + (T_0 - T_1) \operatorname{erf} \left(\frac{x}{2\sqrt{\alpha t}} \right) \quad \text{for } t \geq 0 \quad (11)$$

where erf is the error function defined as

$$\operatorname{erf}(u) = \frac{2}{\sqrt{\pi}} \int_0^u \exp(-\xi^2) d\xi \quad (12)$$

so that $\operatorname{erf}(0) = 0$ and $\operatorname{erf}(u \rightarrow \infty) = 1$ (see appendix IV of [14] for other properties, such as the first derivative formula that we will use below).

From the temperature field given by the above equation one may deduce, by differentiation, the heat flow, $q = \Phi/A$, given by Fourier's law of conduction [14]:

$$q(x, t) = -k \nabla T(x, t) \quad (13)$$

In our case this would lead to

$$q = \frac{k(T_1 - T_0)}{\sqrt{\pi \alpha t}} \exp \left(-\frac{x^2}{4\alpha t} \right) \quad (14)$$

or, at $x = 0$, using definition (8)

$$q = \frac{\varepsilon(T_1 - T_0)}{\sqrt{\pi t}}. \quad (15)$$

This heat flow is not proportional to the thermal conductivity of the material, as under steady state conditions (see equation (2)), but to its thermal effusivity [7, 14, 15].

Suppose now that one brings two half infinite materials with temperatures T_1 and T_2 ($T_1 > T_2$) into ideal thermal contact at $t = 0$. The mutual contact interface acquires a contact temperature T_c in between. This means that the surface temperature of each material becomes T_c . This temperature follows from equation (15) by requiring that the flow out of the hotter surface equals that into the cooler one. One finds

$$\frac{\varepsilon_1}{\sqrt{\pi t}}(T_1 - T_c) = \frac{\varepsilon_2}{\sqrt{\pi t}}(T_c - T_2) \quad (16)$$

giving

$$T_c = \frac{\varepsilon_1 T_1 + \varepsilon_2 T_2}{\varepsilon_1 + \varepsilon_2}. \quad (17)$$

According to this result, if $\varepsilon_1 = \varepsilon_2$, T_c lies halfway between T_1 and T_2 , while if $\varepsilon_1 > \varepsilon_2$, T_c will be closer to T_1 and if $\varepsilon_1 < \varepsilon_2$, T_c will be closer to T_2 . (It is worth noting that the real value of the contact temperature can be affected due to changes in surface characteristics such as roughness.)

The above result is the reason why when we touch bodies of equal temperature but of different effusivities, they do not seem to be equally hot or cold, as explained in our recent work [16]. The contact (feeling) temperature is a function of the effusivity of the body we

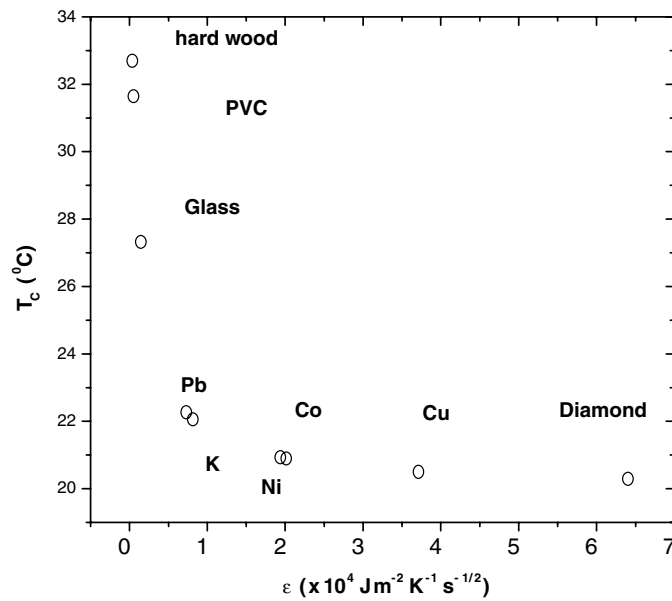


Figure 1. Contact or feeling temperature between human skin at 37 °C and different bodies at 20 °C as a function of their thermal effusivities (after [16]).

Table 1. Thermal properties of different homogeneous solids at room temperature.

Material	K ($\text{W m}^{-1} \text{K}^{-1}$)	α ($\times 10^6 \text{ m}^2 \text{ s}^{-1}$)	ε ($\text{J m}^{-2} \text{K}^{-1} \text{ s}^{-1/2}$)	ρc ($\times 10^{-6} \text{ J m}^{-3} \text{K}^{-1}$)
Diamond	2300	1290	64 040	1.78
Cu	400	116	37 140	3.45
K	102	158	8 150	0.65
Co	100	24.6	20 150	4.05
Ni	91	23	19 400	3.95
Pb	35	23	7 300	1.52
Glass	1.11	0.56	1 480	1.98
PVC	0.20	0.15	515	1.33
Hard wood	0.16	1.77	380	0.09
Human skin	0.37	0.109	1 120	3.39

touch. In figure 1, we show the calculated contact temperature (equation (17)) between human skin at 37 °C and different bodies at 20 °C as a function of their thermal effusivities (see table 1 for values). When touching the metal (e.g. Cu) object of the example mentioned above, as $\varepsilon_{\text{metal}} \gg \varepsilon_{\text{skin}}$, the temperature of the skin drops suddenly to 20 °C, and one senses the object as being ‘cold’. On the other hand, when touching the wooden object ($\varepsilon_{\text{wood}} < \varepsilon_{\text{skin}}$) the skin temperature remains closest to 37 °C, and one senses the object as being ‘warm’. This is also the reason why human foot skin feels the temperature of floors of different materials differently, which are at the same room temperature. If the above experiment is performed on a cold winter day, i.e., the metal and wooden rules are both at the ambient temperature of say 5 °C, then no appreciable differences in their temperature can be found: as the reader can easily see with a simple calculation, equation (17) leads to approximately the same contact temperature for all the materials listed in table 1.

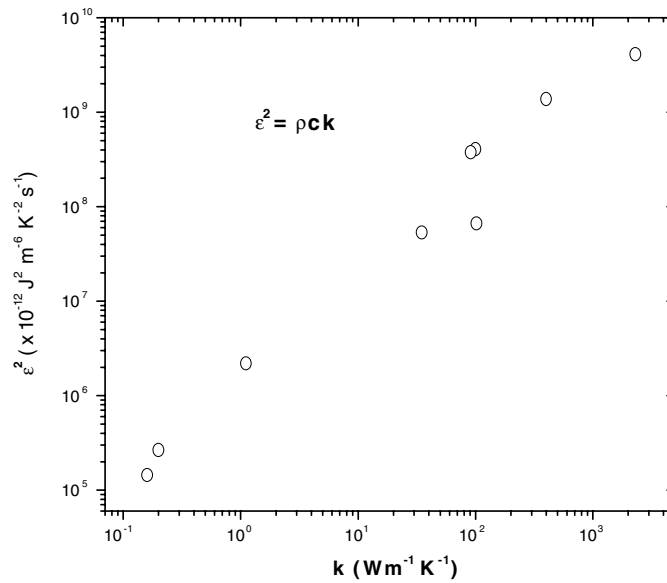


Figure 2. ϵ^2 as a function of k for different solids, showing the proportionality existing between both magnitudes.

In other words, when we touch objects and feel how warm or cool they are, we are dealing with a transient phenomena, which can be interpreted using the thermal effusivity definition. In such a problem it is not only the thermal conductivity that comes into play; the specific heat capacity (ρc) must also be considered, and both are related through the thermal effusivity definition, which becomes the relevant parameter. (After a long enough period of time, we will feel both objects at the same temperature, because our hand, at a higher one, will reach the equilibrium temperature according to the second law of thermodynamics.) In this sense, we have referred above to T_c as the *filling* temperature. Another illustrative example is as follows: consider the situation in which our hand remains at 37 °C but the touched object is at 200 °C; the value of the ‘filling’ temperature then becomes, after equation (17), near 40 °C. This is the reason why our skin does not burn when we make a sudden (transient) contact to a hotter object (but we feel that the object is indeed hotter). For longer contact times, for which there is no longer a transient situation, this experiment will be obviously dangerous for the skin. The source of the common mistake is the coincidence that in solids, a high effusivity material is also a good heat conductor, as figure 2 shows, where ϵ^2 is plotted as a function of k for a variety of homogeneous materials whose properties are given in table 1. One can see that all points are placed close to a straight line because the heat capacity per unit volume, i.e. the product ρc , is almost constant in condensed matter at room temperature (see table 1), as was commented in section 3. (Note that equation (8) can be written as $\epsilon^2 = \rho ck$.)

4. Thermal wave physics

When explaining the thermal effusivity concept, one also has the opportunity to introduce students to the fascinating field of thermal wave physics [17, 18]. Thermal waves are generated in a solid as a consequence of periodical heating. When a thermal wave propagates between two media, the thermal effusivity determines the value of the reflection and transmission

coefficients at the interface, which are defined in a way analogous to that for electromagnetic waves by substituting the refraction index by the thermal effusivity [1, 8]. The wave treatment of heat propagation has been used recently to demonstrate experimentally the occurrence of phenomena that may be regarded as reflection and interference effects in PT experiments [9–12]. In what follows we will review briefly the fundamental ideas beyond this topic.

4.1. Historical remarks

As described in section 1, in photothermal techniques thermal waves are generated in a given sample by means of a periodically varying heat source. Although Ångström in 1861 proposed a temperature-wave method for measuring the thermal diffusivity of a solid in the form of a rod [17], it was not until the 1970s that practical applications of PT techniques concerning the thermal characterization of solids appeared, motivated mainly by the works of Rosencwaig [18, 19] in the emerging field of photoacoustic spectroscopy (PAS), a technique based on the photoacoustic (PA) effect, discovered approximately one century before by Bell [20], and investigated by relevant scientists of the time such as Röntgen [21], Tyndall [22] and Rayleigh [23]. Consequently, with the development of PAS and other PT techniques, the use of wave treatment of heat dates from the 1980s [24], although the concept of the thermal wave first appeared about hundred years before in the works of Fourier [1]. He, as well as Poisson, used equations identical to those used today in describing thermal waves to estimate the thermal properties of the earth's crust, making use of daily periodical temperature oscillations [25]. In the work referred to above, Fourier stated that ‘The problem of temperature at the earth crust presents one of the most beautiful applications of the theory of heat’.

In what follows, we will briefly present a mathematical description of thermal waves, before we describe an experiment based on Poisson's and Fourier's proposals.

4.2. The mathematics of thermal waves

Our starting point will be again the heat diffusion equation. Consider an isotropic homogeneous semi-infinite solid, whose surface is heated uniformly (in such a way that the one-dimensional approach used in what follows is valid) by a light source of periodically modulated intensity, $I_0(1 + \cos(\omega t))/2$, where I_0 is the intensity of the light source, ω is the angular modulation frequency and t is the time. The temperature distribution $T(x, t)$ within the solid can be obtained by solving the homogeneous heat diffusion equation (5), which will be written again as

$$\frac{\partial^2 T(x, t)}{\partial x^2} - \frac{1}{\alpha} \frac{\partial T(x, t)}{\partial t} = 0, \quad x > 0, \quad t > 0, \quad (18)$$

with the boundary condition

$$-k \left. \frac{\partial T(x, t)}{\partial x} \right|_{x=0} = \text{Re} \left[\frac{I_0}{2} \exp(i\omega t) \right], \quad (19)$$

where $i = (-1)^{1/2}$ is the imaginary constant.

This condition expresses that the thermal energy generated at the surface of the solid (for example by the absorption of light) is dissipated into its bulk by diffusion.

The solution of physical interest of the problem for applications in PT techniques is the one related to the time-dependent component. If we separate this component from the spatial distribution, the temperature can be expressed as

$$T(x, t) = \text{Re}[\Theta(x) \exp(i\omega t)]. \quad (20)$$

Substituting in equation (21), we obtain

$$\frac{d^2\Theta(x)}{dx^2} - q^2\Theta(x) = 0, \quad (21)$$

where

$$q = \sqrt{\frac{i\omega}{\alpha}} = (1+i)\sqrt{\frac{\omega}{2\alpha}} = \frac{(1+i)}{\mu} \quad (22)$$

and

$$\mu = \sqrt{\frac{2\alpha}{\omega}}. \quad (23)$$

The general solution of equation (21) using condition (22) has the form

$$\Theta(x) = \frac{I_0}{2kq} i \exp(-qx) = \frac{I_0}{2\varepsilon\sqrt{\omega}} \exp\left(-\frac{x}{\mu}\right) \exp\left[-i\left(\frac{x}{\mu} + \frac{\pi}{4}\right)\right]. \quad (24)$$

From equations (20) and (24) we can obtain the temperature variations $T(x, t)$ as

$$T(x, t) = \frac{I_0}{2\varepsilon\sqrt{\omega}} \exp\left(-\frac{x}{\mu}\right) \operatorname{Re} \left\{ \exp\left[-i\left(\frac{x}{\mu} + \omega t + \frac{\pi}{4}\right)\right] \right\}. \quad (25)$$

It is worth noting that the thermal effusivity of the sample appears in the denominator of the above equation, which has the meaning of a plane wave. Like other waves it has an oscillatory spatial dependence of the form $\exp(-ix/\mu)$, with a wave vector q given by equation (22) and depending on the value of the sample's thermal diffusivity. Because it has several wave-like features, equation (25) represents a thermal or temperature wave of wavelength $\lambda = 2\pi\mu$ propagating with phase velocity [8] given by

$$v_f = \omega\mu = \sqrt{2\alpha\omega}. \quad (26)$$

The parameter μ represents the thermal diffusion length of the thermal wave, i.e. the distance at which the wave amplitude decays e times from its value at the surface of the periodically heated body. As can be seen from equation (23), it depends on the thermal diffusivity and on the modulation frequency. Equation (25) represents, therefore, an attenuated wave. Between the light excitation and the thermal response of the sample there is a phase lag given by the term $(x/\mu + \pi/4)$ in the complex exponent. Several other aspects of thermal wave physics were discussed recently in more detail by Marín *et al* [5, 26].

Note that the thermal-wave behaviour depends on the values of both thermal effusivity, which determines the wave amplitude at $x = 0$, and the thermal diffusivity, on which the attenuation and wave velocity depend. Salazar [15] has given an excellent description of the physical significance of this parameter. A simple experiment based on thermal-wave propagation in soils and the role of thermal diffusivity in this process was recently proposed by McIntosh and Sharratt [27]. Measuring $T(x, t)$ as a function of time, t , at different distances, x , from the earth's surface and fitting the results to equation (25) allow one to determine parameters such as the period of the temperature oscillations, the soil's thermal diffusion length and its thermal diffusivity, as proposed earlier by Fourier. In our laboratory, such an experiment was performed and a device was constructed for automatic measurements of the daily time–air temperature variations as well as of the daily time–temperature variations at different depths beneath the soil surface [28]. In what follows we will try to give a more detailed description of its importance in thermal-wave physics.

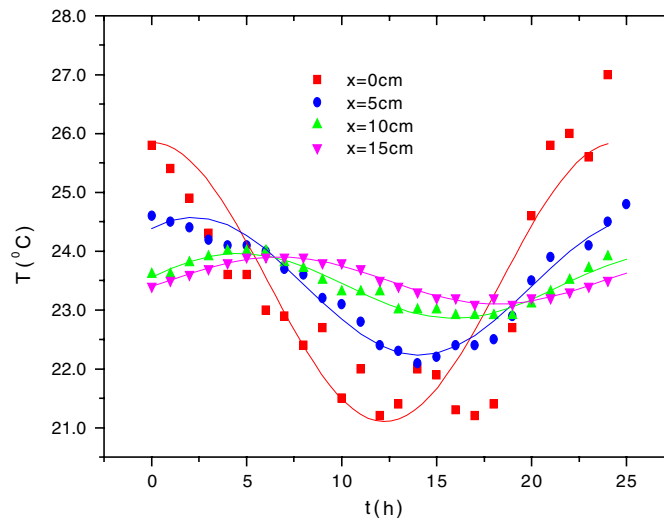


Figure 3. Soil temperature as a function of time at different depths beneath the earth's surface and hourly air temperature. The solid curves represent best fits to equation (25). The air temperature ($x = 0$) versus time curve is also represented for comparison. In the figure, 0 h corresponds to the 12 PM, 5 h to 17 h, 10 h to 22 h and so on (taken from [28]).

(This figure is in colour only in the electronic version)

4.3. One experiment based on thermal wave propagation in soils

The surface of the earth is exposed to more or less regular daily temperature fluctuations, owing to different causes, that can differ appreciably from day to day, from season to season and from region to region. As a consequence, air and soil temperatures generally exhibit a diurnal cycle with a period $T = 2\pi/\omega$ of near 24 h. The temperature variation $T(x, t)$ with time, t , and with depth below the surface, x , is given by equation (25). Measuring $T(x, t)$ as a function of time, t , at different distances, x , from the earth's surface and fitting the results to equation (25) allow one to determine parameters such as the period of the temperature oscillations, the soil's thermal diffusion length and its thermal diffusivity, as proposed earlier by Fourier.

We have performed measurements using LM-335 solid-state temperature sensors incorporated into a computer-controlled probe. In figure 3, we show the hourly soil temperature measured at various depths beneath the soil surface during 24 h on 6 March 2002, a typical clear winter day in Havana, Cuba, as well as the hourly air temperature at the same location [28]

In the figure we can see the phase shift and the attenuation of the temperature waves with depth, as well as their dependence on soil thermal properties, in particular, its thermal diffusivity. One can easily see the attenuation of the temperature wave (i.e. the reduction in the wave amplitude) as well as the phase shift between the maximum and minimum values of the curves with depth. From the fits the value obtained for the angular frequency is $\omega = (0.256 \pm 0.008) \text{ h}^{-1}$. One can observe from the figure that the variation in the air temperature is approximately sinusoidal, with the period $T = 2\pi/\omega = (24.54 \pm 0.03) \text{ h}$, near to the expected one of 24 h. From the mean value of the thermal diffusion lengths, the soil's thermal diffusivity can be calculated using equation (23). We have obtained $\alpha = (0.0022 \pm 0.0008) \text{ cm}^2 \text{ s}^{-1}$, in agreement with previous reported values [29]. Using this experiment

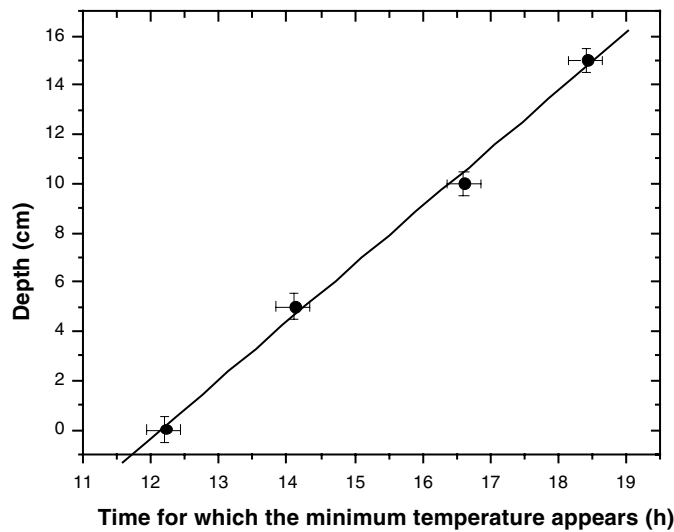


Figure 4. Depth at which the measurements were performed as a function of the time at which the minimum temperature appears. The solid line is the best linear fit of the data (taken from [28]).

we have developed several exercises to introduce undergraduate students of physics and engineering to the very fascinating fields of thermal wave and soil physics, which are described in more detail elsewhere [28]. For example, in order to describe in a more quantitative way the phase shift of the temperature oscillations with depth, we show in figure 4 the relationship between the time at which the minimum temperature appears and the depth at which the measurements were performed. The slope of the straight line represents the phase velocity. From the linear fit of the data we have obtained the value $v_f = (2.4 \pm 0.1) \text{ cm h}^{-1}$, which is similar to the value that one can obtain from the phase velocity definition, equation (26), using the values of ω and μ obtained from the fits represented in figure 3.

Because the experiment described is computer controlled, students gain valuable experience using research-grade measurement control, data acquisition and data analysis software. It is possible within one laboratory session to program the measurement, whose results can be collected the next day in electronic format for data analysis and processing. In addition, soil is an easily accessible physical system of great importance due to the role that it plays in our food, shelter and well-being.

4.4. Behaviour at interfaces

As with other kinds of waves, thermal waves experience reflection and refraction. The characteristic coefficients for these phenomena are determined by the thermal effusivity values of the involved media from the thermal wave equation (25). When thermal waves propagate between the surface of two media with different thermal effusivities, then they interfere in the same way as electromagnetic, acoustic or mechanical waves do, a behaviour that can be exploited in numerous applications [9–12, 30–34].

4.4.1. Reflection and refraction of thermal waves. Consider two regions, 1 and 2, separated by the plane $x = 0$. Suppose that the incident, reflected and transmitted plane thermal waves make angles θ_i , θ_r and θ_t with the x axis, respectively. One can show that after some straightforward

calculations the reflection and transmission coefficients can be written as [2]

$$R = \frac{\cos(\theta_i) - b \cos(\theta_t)}{\cos(\theta_i) + b \cos(\theta_t)} \quad (27)$$

and

$$T = \frac{2 \cos(\theta_i)}{\cos(\theta_i) + b \cos(\theta_t)}, \quad (28)$$

where

$$b = \frac{\varepsilon_1}{\varepsilon_2} \quad (29)$$

is the ratio of the media thermal effusivities. Thermal effusivity can be regarded, therefore, as a measure of the thermal mismatch between the two media, playing a role similar, for example, to the refraction coefficient in similar optical phenomena.

It is important to note that for normal incidence ($\theta_i = \theta_t = 0$) one can show that the above coefficients reduce to

$$R = \frac{1 - b}{1 + b} \quad (30)$$

and

$$T = \frac{2}{1 + b}. \quad (31)$$

Several applications of these results can be seen, for example, in the book of Almond and Patel [2], where other thermal wave effects are also described, such as scattering at finite boundaries and objects. In the next section, we will present an experiment demonstrating how the above results can be applied to describe the propagation of thermal waves through a medium limited by two thermal ‘mirrors’ experiencing interference.

4.4.2. An experiment with thermal waves: interference. Thermal wave interference was first discussed by Bennett and Patty [9], and then explored by Shen and Mandelis [10], who succeeded in demonstrating the feasibility of the pyroelectric detection of thermal waves propagating across an air gap formed between a pyroelectric temperature sensor and a foil of another material (i.e. aluminium) acting as the source of the thermal waves. More recently, the usefulness of this detection scheme for the monitoring of mass diffusion in air was demonstrated [11, 12], as well as its capability of performing the thermal characterization of gases [30], liquids [31] and their mixtures [32–34]. The potential use of this experimental configuration for fuel analysis has been recognized elsewhere [35].

In figure 5 we show schematically a typical experimental set-up. It consists of a cavity of variable length L , denoted formerly thermal wave resonator [10] and then thermal wave interferometer (TWI) [30–33]. The cavity, filled for example by air, is formed between a thin (15 μm thick) Al foil and a pyroelectric sensor (25 μm thick polyvinylidene difluoride (PVDF) film with Al metallized surfaces), both 5 mm in diameter. A laser light beam modulated at 10 Hz by means of a mechanical chopper, impinges on the black-painted outer surface of the Al foil, which acts as a light absorber. Following the absorption of the modulated light beam, the Al foil temperature fluctuates periodically at the modulation frequency of the incident beam thereby launching a thermal wave into the gas-filled cell. The thermal waves thus generated propagate back and forth between the Al foil and the pyroelectric sensor separated by a distance L . On striking the gas–Al and gas–sensor boundaries, the thermal waves are partially reflected, and interference between the reflected and incident wave trains will set in. As discussed elsewhere [9–12, 30–34] in more detail, the temperature rise resulting from the

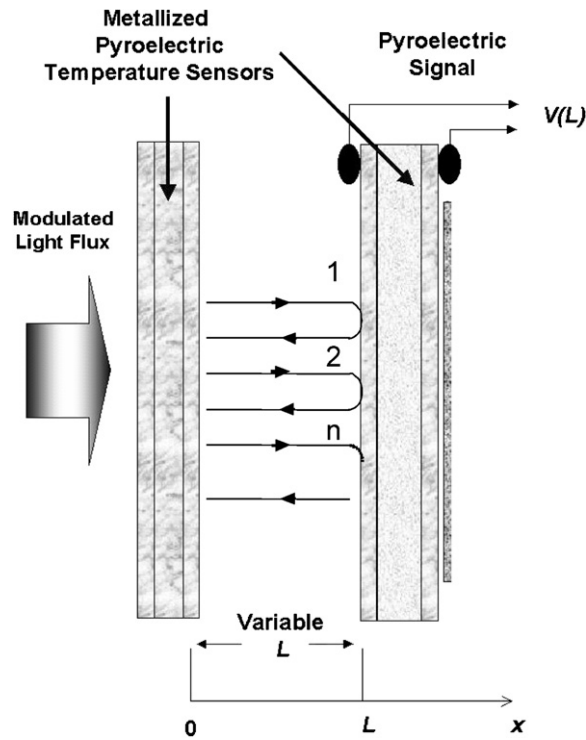


Figure 5. Schematic view of the thermal wave interferometer described in the text, showing the principles of thermal-wave interferometry (taken from [32]).

superposition of all arriving waves can be measured at the pyroelectric surface as a function of the gas-layer thickness.

The temperature distribution $T(x, t)$ within the gas region along the longitudinal x coordinate following the periodic heating of the Al foil, can be obtained by solving the heat diffusion equation with the boundary condition that the heat generated at the solid surface by light absorption is dissipated in the gas by diffusion. Accordingly, this solution is similar to equation (25), i.e [11],

$$T(x, t) = \frac{I_0}{2\varepsilon_g\sqrt{\omega}} \exp(-qx) \exp(i\omega t) = \frac{I_0}{2\varepsilon_g\sqrt{\omega}} \exp(-x/\mu) \exp[-i(x/\mu - \pi/4 - \omega t)], \quad (32)$$

where ε_g is the thermal effusivity of the gas, $q = (i\omega/\alpha_g)^{1/2}$, α_g is the gas thermal diffusivity and $\mu = (2\alpha_g/\omega)^{1/2}$ is the characteristic thermal wave diffusion length in the gas.

Consider now the propagation of a plane thermal wave described by equation (32) through the gas medium (denoted g) contained between the media A (corresponding to the Al foil) and P (corresponding to the PVDF sensor). The temperature at the surface $x = L$ is obtained by summing all the waves arriving at this point in the following way (for sake of simplicity, from now on we shall omit the term $\exp(i\omega t)$ in our calculations):

$$T(L) = T_1 + T_2 + \dots + T_n = T_0 [e^{-qL} + R_{gA}R_{gP} e^{-3qL} + \dots + (R_{gA}R_{gP})^n e^{-(2n+1)qL}], \quad (33)$$

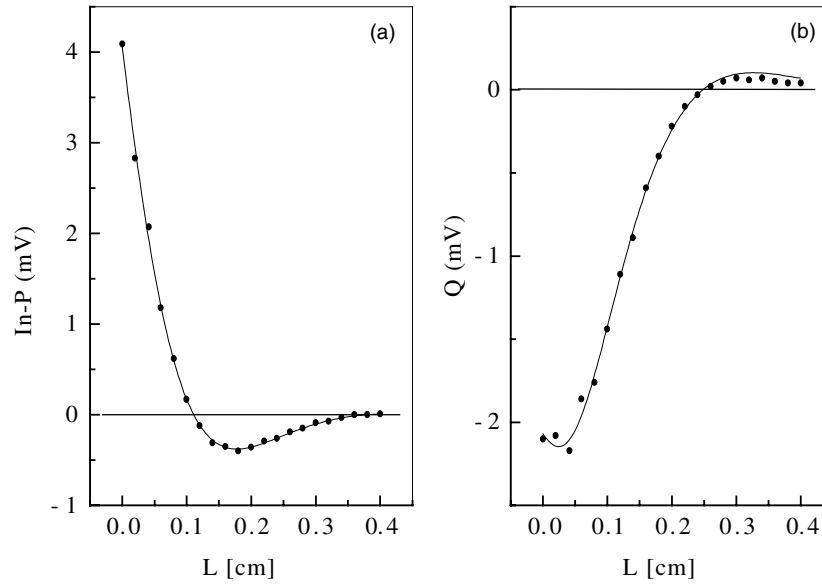


Figure 6. (a) In-phase and (b) quadrature signals measured for air. The solid curves are the best-fit results to the real and imaginary parts of equation (34) respectively (taken from [11]).

where $T_0 = I_0/(2\varepsilon_g\omega^{1/2})$ is the gas temperature at $x = 0$, and R_{gA} and R_{gP} are the reflection coefficients at the interfaces gas–Al and gas–PVDF, respectively, as given by equation (30). Summing up the geometric series in equation (33), one gets

$$T(L) = \frac{T_0 \exp(-qL)}{1 - \gamma \exp(-2qL)}, \quad (34)$$

where we have $\gamma = R_{gA}R_{gP}$. The temperature rise given by equation (34) induces an electric field between the two opposite surfaces of the PVDF sensor. The real (in-phase) and imaginary (quadrature) parts of the resulting voltage, V , as well as its amplitude and phase, can be determined by lock-in based detection [36] as a function of the cavity length, L .

In figure 6, we present results obtained [10] for the air-filled cavity at ambient pressure and temperature. Figure 6(a) shows the in-phase (In-P) signal as a function of the cavity length (L), whereas in figure 6(b) the quadrature (Q) signal versus L is plotted. The solid and dashed lines represent respectively the best fit of the experimental data to the real and imaginary parts of the voltage given by the sensor, which is proportional to the temperature expressed by equation (34). The thermal diffusivity was taken as an adjustable parameter. The obtained value for this parameter was $\alpha = (0.244 \pm 0.007) \text{ cm}^2 \text{ s}^{-1}$, which is in good agreement with literature reported values [37], showing the validity of the thermal wave interference approach.

The experiment described can be useful in familiarizing students with thermal-wave physics, as well as with modern measurement techniques and effects, such as the lock-in and the photopyroelectric one. As measurements are performed at a single modulation frequency a simple and low-cost lock-in instrument can be constructed and used for this purpose [38]. The rest of the apparatus is also easily available. A low intensity HeNe laser is common in today's physics laboratories as well as a micrometer stage. Pyroelectric sensors are commercially available at low cost in the form of sheets of different thicknesses (a pyroelectric crystal can also be used). Because such an experiment can also be controlled by a computer, time that

is spent doing routine data collection can instead be used to perform a greater number of measurements.

4.4.3. Analogy with electrical phenomena: thermal effusivity as thermal impedance. The use of analogies is very common in physics. We have discussed above how the temperature oscillations generated by periodical heat sources can be interpreted on the basis of their analogy with waves. In this section we will briefly discuss how the well-known analogy between electrical and thermal phenomena is useful for interpreting the meaning of thermal effusivity. This analogy shows that temperature differences are equivalent to potential differences (voltage) and heat flux plays the role of electrical current (see equation (3)). Then, following the definition of electrical impedance, Z , as a ratio of the potential difference between two points of a conductor and the flowing electrical current ($Z = V/I$), the thermal impedance, Z_t , can be defined as the temperature difference between two faces of a thermal conductor divided by the heat flux crossing the conductor. If we have a periodic oscillating temperature field (thermal waves), then the thermal impedance becomes the ratio between the change in thermal-wave amplitude and the thermal-wave flux. At the surface of the semi-infinite medium treated as above, we get

$$Z_t = \frac{\Theta(x=0, \omega) - \Theta_0}{-k \frac{d\Theta(x, \omega)}{dx} \Big|_{x=0}}, \quad (35)$$

where Θ_0 is the ambient (constant) temperature and $\Theta(x, \omega)$ the spatial part of the thermal wave field. Substituting equation (24) into equation (35), one obtains

$$Z_t = \frac{1-i}{\varepsilon\sqrt{\omega}} = \frac{1}{\varepsilon\sqrt{\omega}} \exp\left(-i\frac{\pi}{4}\right). \quad (36)$$

A material with a low thermal effusivity presents higher thermal impedance than that with a higher value, having therefore a greater resistance to the flow of heat. This result is also in agreement with the behaviour shown in figure 2, where the direct proportionality between thermal effusivity (better, say, its square) and conductivity is represented. The role of thermal impedance at interfaces between composite media is discussed in detail in a recently published book by Mandelis [39]. An experiment describing the application of the thermal impedance concept for measurement of thermal effusivity is reported by Defer *et al* [40].

5. Concluding remarks

In this paper we have tried to clarify the role played by thermal properties in heat transfer phenomena. Although the four fundamental thermal parameters are related to each other by means of equations (6) to (8), each of them has its own meaning. While static and stationary phenomena are governed by parameters such as specific heat and thermal conductivity respectively, under non-stationary conditions thermal effusivity and diffusivity are the more important ones. The former plays a fundamental role in the case of a transient phenomenon in which a body is placed suddenly in contact with another one: the body with a higher effusivity extracts more heat than that with a lower value of this parameter. On the other hand, it is, together with thermal diffusivity, one of the most important thermophysical parameters when we deal with periodical time-varying processes. Both parameters play a very important role in the expanding field of thermal-wave physics and techniques, where the former determines the amplitude of the thermal-wave fields and describes their behaviour at interfaces between different media, and thermal diffusivity determines the attenuation of the thermal waves with depth for a given modulation frequency and wave propagation velocities. In particular, the

role of thermal effusivity is usually undervalued or misunderstood, and can be exploited in advanced introductory physics courses, especially at high-school level, where the analysis of heat transport problems presented in standard textbooks does not make any use of the concept. However, the simple experiments described here demonstrate that we are in reality very familiar with this parameter. These experiments can be performed in student laboratories to illustrate the concepts discussed.

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