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LETTERS AND COMMENTS

On thermal waves

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Abstract
This letter explains briefly why so-called thermal waves are not truly waves—despite being described mathematically as such—because of their intrinsic diffusive character. This topic should be of interest to those students and teachers dealing with heat transport in the presence of non-stationary harmonic heat sources, photothermal phenomena and techniques, and other related subjects.

It is well-established that Fourier's law (FL) of heat conduction violates relativity theory, despite describing many daily phenomena well [1]. This is because it assumes instantaneous propagation of thermal signals. In reality, when a physical system is exposed to a temperature gradient, $\vec{\nabla}T$, the flux of heat is not established instantaneously as FL predicts, but only after a characteristic time, τ, has elapsed.

The equation that relates the heat flux density, $\vec{j}$, with $\vec{\nabla}T$ is

$$\vec{j}(\vec{r}, t+\tau) = -K\nabla T(\vec{r}, t), \qquad (1)$$

where K is thermal conductivity.

Because τ is small (otherwise FL does not work in ordinary circumstances) one can expand the first term of equation (1) in Taylor series around $\tau = 0$. The result is

$$\vec{j}(\vec{r}, t+\tau) = \vec{j}(\vec{r}, t) + \tau\frac{\partial \vec{j}(\vec{r}, t)}{\partial t} + \ldots = -K\nabla T(\vec{r}, t). \qquad (2)$$

One can see that for $\tau = 0$ equation (2) leads to

$$\vec{j}(\vec{r}, t) = -K\nabla T(\vec{r}, t). \qquad (3)$$

This is the common FL that leads, when combined with the energy conservation law (continuity equation), to the parabolic heat diffusion equation that describes non-stationary heat phenomena.

One can see that if $\tau \neq 0$ is very small then one has to work with equation (1). Neglecting higher order terms, equation (2) leads to

$$\vec{j}(\vec{r}, t) + \tau\frac{\partial \vec{j}(\vec{r}, t)}{\partial t} = -K\nabla T(\vec{r}, t). \qquad (4)$$

Equation (4) is known as Cattaneo's law [2], which together with the continuity equation leads to the hyperbolic heat diffusion equation (HHDE)

$$\nabla^2 T - \frac{1}{D}\frac{\partial T}{\partial t} - \frac{\tau}{D}\frac{\partial^2 T}{\partial t^2} = 0, \tag{5}$$

where D is the thermal diffusivity. Note that if $\tau = 0$ it becomes the well-known parabolic heat diffusion equation.

The HHDE is unfamiliar to many people, even to some researchers involved in heat transfer problems. Its solution in the presence of harmonic heat sources modulated at a given frequency, f, is of particular importance. Although it is different for low and high modulation frequencies [3] (i.e. for $f \ll \tau^{-1}$ and $f \gg \tau^{-1}$, respectively), it has the mathematical form of a damped temperature oscillation [3, 4]. Thus it is often called a 'thermal wave' (or a 'heat wave' by some authors [5]), a term that has motivated many discussions about whether thermal waves are truly waves. Several answers have been given. One of the most widely accepted explanations was given by Salazar in this journal [3], in which he demonstrated that thermal waves are only oscillations of the temperature field, they not transport energy as real waves must.

To support this explanation further one can simply look at the similarity between equation (5) and the equation describing the propagation of electromagnetic (real) waves in electrical conductive media:

$$\nabla^2 \vec{E} - \sigma\mu\frac{\partial \vec{E}}{\partial t} - \varepsilon\mu\frac{\partial^2 \vec{E}}{\partial t^2} = 0. \tag{6}$$

Here E is the electric field, σ is the electrical conductivity, ε is the electrical permittivity and μ is the magnetic permeability.

It is well known that in wave equations of this kind the first derivative controls the damping and the second governs the propagation velocity of the solutions. In electromagnetic waves the damping depends on electrical conductivity. If $\sigma = 0$, there is no damping and the medium becomes transparent to a wave propagating with the speed of light $c = (\varepsilon\mu)^{-1/2}$. But in the case of thermal waves the situation is quite different because the thermal diffusivity, D, appears in both derivatives (see equation (5)), so that it controls both the damping and the wave velocity. Only for an infinite diffusivity will there be no thermal wave damping. But in this hypothetical case the wave velocity, $u = (D/\tau)^{1/2}$ would be infinite too, and this makes no physical sense [4, 6].

For this reason one can conclude that 'thermal waves', although described mathematically as waves, do not behave like ordinary waves. They have an intrinsic diffusive character determined by the thermal diffusivity value. However, it is surprising that the results of many experiments involving thermal waves (e.g. photothermal techniques [7]) can be explained using models based in phenomena like reflection, transmission and the interference of waves [8, 9]. Thus, the debate on this topic is not settled and new experiments are needed to understand it better. This work is an attempt to motivate further reflection and discussion on this topic, not only among specialists in heat transport, photothermal techniques and related areas of research, but also undergraduate and graduate students of physics and engineering.

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References

[1] Marín E 2011 *Latin-Am. J. Phys. Educ.* **5** 402
[2] Cattaneo C 1948 *Atti Semin. Mat. Fis. Univ. Modena* **54** 3
[3] Salazar A 2006 *Eur. J. Phys.* **27** 1349
[4] Marín E, Marín J and Díaz P 2002 *Eur. J. Phys.* **23** 523
[5] Joseph D D and Preziosi L 1989 *Rev. Mod. Phys.* **61** 41
[6] Salazar A 2006 Personal Communication
[7] Almond D P and Patel P M 1996 *Photothermal Science and Techniques (Physics and its Applications* vol 10*)* ed E R Dobbsand and S B Palmer (London: Chapman and Hall)
[8] Marin E 2007 *Eur. J. Phys.* **28** 429
[9] Marín E, Vera-Medina G, García A and Calderón A 2010 *Cent. Eur. J. Phys.* **8** 634